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ABSTRACT

Based on a Monte Carlo simulation, this study is designed to investigate the power of the Kruskal-Wallis's H-test compared to the power of the F-test for three equal moderate sample sizes drawn at random from distributions of common or different shapes but for which the population distributions have equal variances. The distributions are the Normal, Uniform, and Double Exponential. It was found that the F-test is robust to violating assumptions of non-normality for sample of size 10, 15, and 20 but the power of H-test is affected by the shape of the population distributions. The power of the H-test increase faster when all samples are drawn from Double Exponential distribution than the power of H-test drawn from all Normal or all Uniform distribution. It is also found that the power of H-test is greater than the power of the F-test when all samples are drawn from double exponential distribution and the combinations of double exponential and normal distributions. The power of H-test is almost identical to the power of F-test when 2 samples are drawn from the double exponential and one sample is from the uniform distribution and when 3 samples are drawn from 3 different shaped distributions. The power of H-test is less than the power of the F-test when all samples are from normal or from uniform distributions. (Author/BB)

MONTE CARLO STUDY OF THE POWER OF H-TEST COMPARED TO F-TEST WHEN POPULATION DISTRIBUTIONS ARE DIFFERENT IN FORM

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This study, based on a Monte Carlo simulation, is designed to investigate the power of the Kruskal-Wallis's H-test compared to the power of the F-test for three equal moderate sample sizes drawn at random from ditributions of common or different shapes but for which the population distributions have equal variances. The distributions are the Normal, Uniform, and Double Exponential. It was found that the F-test is robust to violating assumptions of non-normality for sample of size 10, 15, and 20 but the power of H-test is affected by the shape of the population distributions. The power of the H-test increase faster when all samples are drawn from Double Exponential distribtion than the power of H-test drawn from all Normal or all Uniform distribution. It is also found that the power of H-test is greater than the power of the F-test when all samples are drawn from double exponential distribution and the combinations of double exponential and normal distributions. The power of H-test is almost identical to the power of F-test when 2 samples are drawn from the double exponential and one sample is from the uniform distribution and when 3 samples are drawn from 3 different shaped distributions. The power of H-test is less than the power of the F-test when all samples are from normal or from uniform distributions The power of the H-test is also less than the F-test when 2 samples are drawn from uniform and the third is from the normal or exponential distribution.

Introduction

The power of test is a means for comparing two methods proposed to test the same hypothesis. Relative Efficiency is one procedure used in comparing one test with a second test. It refers to the ratio of two numbers of observations required by each test to have equal power for the same probability of Type 1 error. By far the most common index of efficiency is expressed in terms of Assymptotic Relative Efficiency (A.R.E.) which indicates the limiting value of the ratio of observations when one value approaches infinity. The A.R.E. often provides a compact summary of relative efficiency between two tests but its use is limited since it considers infinite sample sizes. However, the analytical comparison between two tests is usually found in terms of A.R.E. figures. The A.R.E.'s of non-parametric tests compared to their related parametric tests are usually reported for normally distributed population with few indexes reported for the other conditions (see Bradley,1968, p 61). Even the comparison of the Wilcoxon rank sum test with the Normal Scores test is also found under normal distribution theory. Lehman (1953, p 23) states that:

> ... when comparing different rank tests, one is no longer tied to normal alternatives, but it would on the contrary seem rather desirable to make comparisons in terms of non-parametric classes of alternatives.

It would seem that more useful information would be available to the practitioner if power comparisons were made under weaker conditions, such as test A has greater power than test B when used for morderate sample sizes drawn from a symmetric distribution. When a comparison, such as the two sample Wilcoxon rank sum test compared to the two sample t-test, is made in terms of A.R.E. under normal condition it seems as if the models are being compared in a general situation but in reality the information refer to infinite sample sizes. Thus, the comparison is not useful to the practitioners since infinite samples are never selected.

The Kruskal-Wallis's H-test is the K-sample generalization of the Wilcoxon two-sample test and may be described as the rank analog of the single-classification analysis of variance test. The A.R.E. of H-test relative to F-test is $\pi/3$ or .955 in situation where assumptions met for the F-test are satisfied. Hodges and Lehman (1956) proved that if the distribution functions have identical shape but differ only in location, theA.R.E. of H-test relative to F-test is never less than .864 and may exceed 1 for certain type of distribution (Connover, p203). A.R.E. of H-test to F-test is 1.00 when common shape is rectangular (Bradley). If the underlying distributions are non-normal, the efficiency of rank test relative to that of the parametric test is always at least .864 and is greater than unity for some commonly encountered non-normal distributions such as the uniform distribution and exponential distribution (marascuilo).

The purpose of the present study is to compare the power of the H-test to the power of the F-test for the fixed-effect one-way ANOVA model. The comparison is made under three moderate sample sizes which are drawn from the parent distributions in discrete form.

Three population distributions are the Normal, Uniform, and Double Exponential distributions. The normal distribution is adapted from the table of standard normal scores. The range in X extends from low of 463 to a high of 537. The parameters of the distribution are given by a mean of 500 and a variance of 100.05. The uniform distribution is adapted from the formula $f(X) = 1/(b-a), \quad a \leq X \leq b$. The range is 483 - 517 for the distribution having mean 500 and variance 100.51. The double exponential distribution is obtained from the formula $f(u) = (1/14.14)\, e^{-u/7.07}$ where $-50 \leq u \leq 50$, and then transformed to $X = 500 + u$. The double exponential distribution has mean of 500 and variance of 100.06. Thus, the study is based on approximately equal population variances of 100.

The purpose of the study can be stated specifically by the following questions:

1. What is the power of the H-test compared to the F-test when samples are drawn from a common distribution and
   - 1.1 when all samples are drawn from a normal distribution?
   - 1.2 when all samples are drawn from a uniform distribution?
   - 1.3 when all samples are drawn from a double exponential distribution?
2. What is the power of the H-test compared to the F-test when samples are drawn from different populations with different shapes and
   - 2.1 when two samples are drawn from normal and another one is from a uniform distribution?
   - 2.2 when two samples are drawn from normal and another one is from a double exponential distribution?
   - 2.3 when two samples are drawn from uniform and the third is from normal distribution?
   - 2.4 when two samples are drawn from uniform and the third is from double exponential distribution?

2.5 when two samples are drawn from double exponential and the third is from a normal distribution?

2.6 when two samples are drawn from double exponential and the third is from a uniform distribution?

2.7 when one sample is drawn from a normal, one is from a uniform, and another one is from a double exponential distribution?

3. How does the size of sample affect the power of the H-test compared to the F-test?

4. How does the distance between the means of the distributions affect the power of the H-test relative to the F-test?

Procedure

A CDC 6400 computer was programmed to call uniform random numbers from a subroutine RANF. Let the i th random number be assigned by X(I) the numbers X(I) were then partitioned into three equal groups according to the specified sample sizes NJ. The F-value was computed on the values of the X(I)s but the H-value was computed on the Rank(I) which corresponded to the value X(I). This operation was replicated 1000 times. The computer counted the F-values and H-values which exceeded their critical values for $\alpha = .05$ and then listed them. When the null hypothesis was true the computer listed the approximate probability of a type 1 error. The power of the F-test and the H-test were obtained when the means of the population distribution were different under the same computational scheme. The parameter Delta, δ, was used to indicate the degree of inequality between means and is specified in terms of the standard deviation of the population. With this notation, $\mu_1 = 500 - \delta$, $\mu_2 = 500$, $\mu_3 = 500 + \delta$.

The power of F-test and H-test was investigated for the case of 3 equal sample of size 10, 15, and 20 and for Delta of $.25\sigma$, $.50\sigma$, $.75\sigma$, and 1.00σ.

Results

The empirical power of the H-test compared to F-test, for nominal $\alpha = .05$, is shown in table 1 for a number of conditions. The power of the F-test listed first and the power of the H-test is listed below in parenthesis. The three capital letters listed in the first column denote the shapes of the population distributions fromwhich the samples are drawn. The sizes of the samples are shown on the first row, and the discrepancies between the population means, delta, are indicated on the second row. The pair of numbers, .965 and (.966), reported in row NUE and column 1.0 S.D. for NJ = 10, are the power of the F-test and the H-test when the first sample is from the Normal, the second is from the Uniform, and the third is from the Double Exponential distribution in which the mean of the first distribution is $\mu_1 = 500 - \sigma = 490$, the mean of the second distribution is 500, and the mean of the third distribution is $\mu_3 = 500 + \sigma = 510$. In order to make the presentation more compact, the symbol as ' power of F-(NUE,10,1 and 'power of H-(NUE,10,1.0)' will be used to indicate the pwer of F-test and H-test as mention above.

Table 1 shows, as expected, that the power of F-test and power of H-test increase as delta increases. When the comparison is observed across sample sizes, the powers increase faster when sample size is larger.

On the empirical level of significance.

The empirical level of significance is reported in table 1 for Delta = 0. Since the study is based on 1000 replications, for each cell, then the standard error for the proportion is about .0067. Thus, the 95% confidence interval for empirical α is in the range of .037 to .063 for the nominal α of .05. Therefore, the H-test is conservative when applies to the UUU, and EEE types of population distributions for sample size of 15, and nominal α =.05. This is also true for F-(EEE, 15, 0).

The comparisons of the power of the F-test across the distributions with a common shape are shown in figure 1.1, 1.2, and 1.3. The corresponding comparisons of the H-test are in figure 2.1, 2.2, and 2.3.

The power curves of F-(NNN,10), F-(UUU,10), and F-(EEE,10) are observed in figure 1.1. The shapes of the curves are similar and very close together. The shapes and the discrepancy are still close even when the same size is 15 or 20 as can be observed in figure 1.2 and 1.3. However, the rate of increase in power, along the magnitude of delta, is greater when the sample sizes are increased. These fiegures could lead to the conclusion that F-test is robust for violating the assumption of non-normality.

The comparison of the powers of H-(NNN,10), H-(UUU,10), and H-(EEE,10) does not hold the same character as those of the F-test. Figure 2.1 shows that the power H-(EEE,10) increases faster than power of H-(NNN,10), and also ster than the power of H-(UUU,10). This character is also true for sample

Table 1 Comparison of Actual (Empirical) Power of F-test and H-test when Three equal samples are drawn from a Common and Different shapes of population distribution for various Sample sizes, various Delta, the nominal significance level = .05

	NJ = 10					NJ = 15					NJ = 20		
	Delta					Delta					Delta		
Shape of Distribution	0. S.D.	.25 S.D.	.50 S.D.	.75 S.D.	1.0 S.D.	0. S.D.	.25 S.D.	.50 S.D.	.75 S.D.	1.0 S.D.	.0 S.D.	.25 S.D.	.50 S.D.
N N N	.047	.142	.453	.800	.974	.038	.197	.675	.956	.997	.043	.261	.806
	(.044)	(.129)	(.411)	(.754)	(.963)	(.039)	(.188)	(.633)	(.951)	(.997)	(.039)	(.242)	(.775)
U U U	.049	.136	.441	.801	.975	.040	.204	.661	.964	.998	.050	.244	.794
	(.043)	(.135)	(.371)	(.714)	(.951)	(.033)	(.181)	(.603)	(.927)	(.996)	(.048)	(.221)	(.728)
E E E	.046	.147	.475	.804	.958	.031	.209	.683	.949	.995	.045	.281	.811
	(.043)	(.169)	(.538)	(.846)	(.971)	(.035)	(.266)	(.762)	(.976)	(.998)	(.040)	(.343)	(.884)
N N U	.047	.141	.447	.796	.972	.052	.188	.634	.956	.999	.053	.248	.787
	(.043)	(.127)	(.376)	(.723)	(.961)	(.050)	(.175)	(.568)	(.925)	(.996)	(.050)	(.214)	(.730)
N N E	.048	.143	.464	.804	.967	.054	.202	.632	.950	.997	.050	.252	.806
	(.046)	(.135)	(.455)	(.782)	(.967)	(.053)	(.207)	(.650)	(.953)	(.997)	(.051)	(.245)	(.821)
U U N	.044	.145	.443	.798	.972	.051	.191	.632	.952	.999	.050	.248	.806
	(.044)	(.123)	(.377)	(.729)	(.952)	(.052)	(.172)	(.567)	(.930)	(.996)	(.051)	(.214)	(.748)
U U E	.047	.144	.456	.803	.974	.058	.197	.639	.961	.999	.051	.245	.819
	(.045)	(.117)	(400)	(.757)	(.970)	(.057)	(.166)	(.588)	(.946)	(.999)	(.050)	(.213)	(.770)
E E N	.047	.149	.473	.806	.968	.045	.200	.644	.946	.999	.050	.260	.808
	(.045)	(.152)	(.479)	(.807)	(.968)	(.050)	(.216)	(.682)	(.962)	(.998)	(.048)	(.276)	(.840)
E E U	.048	.145	.466	.803	.967	.046	.201	.643	.948	.998	.050	.248	.799
	(.048)	(.134)	(.428)	(.771)	(.966)	(.050)	(.185)	(.613)	(.946)	(.998)	(.048)	(.222)	(.780)
N U E	.046	.144	.461	.797	.965	.054	.199	.646	.948	.998	.052	.257	.802
	(.044)	(.134)	(.425)	(.765)	(.966)	(.057)	(.188)	(.630)	(.943)	(.998)	(.042)	(.231)	(.803)
Theoretical power* for F-(NNN)	.050	.143	.459	.815	.972	.050	.219	.676	.945	.999	.050	.272	.802

Power of F-test listed first, power of H-test listed below in parenthesis.

N N N denote the first sample is from Normal, the second from Uniform, and the third from Double Exponential distribution.

*From Table of the Power of F-test (Tiku,M.L.,1967) size of 15 and 20 which can be seen in figure 2.2 and 2.3. The shapes of power function for H-(NNN,10), H-(UUU,10), and H-(EEE,10) are not the same. Power of H-(NNN,10) increases faster than H-(UUU,10) for delta in the range of $.05\sigma$ to $.75\sigma$. However, the shape and the discrepancy between power of H-(NNN) and H-(UUU) becomes closer when sample sizes are increased.

The conlcusion can be made at this point that: the power functions of F-test are the same for three equal moderate sample sizes drawing from all normal, all uniform, and all double exponential distribution but the power of H-test obtained from double exponential distribution is greater than those obtained from normal or uniform distributions. The shape and the discrepancy between power of H-(NNN) and H-(UUU) become similar and close toger when sample size is larger.

Figures 3.1, 3.2, and 3.3 show the comparison between the power of F-(NNN) and H-(NNN) for sample sizes of 10, 15, and 20 respectively. The shapes of the power curves are the same within the same sample size. Power of F is greater than power of H for all sample sizes are larger.

The comparison of powers between F-(UUU) and H-(UUU) are shown in figures 4.1, 4.2 and 4.3. The power of F-(UUU) is also greater than H-(UUU) for sample size of 10, 15, and 20. The shapes of F-(UUU,10) and H-(UUU,10) are slightly different. Power of F-(UUU,10) increases faster when delta are in the range $.50\sigma$ to $.75\sigma$, but when sample size is larger, their shapes are more similar and the discrepancy becomes smaller.

Figures 5.1, 5.2, and 5.3 show the comparison of power of F-test and H-test when all samples are drawn from double exponential distribution. It appears that power of H-test is greater than the corresponding F-test for all sample sizes. The shapes of the power curves are similar within the same sample size. However, the discrepancies between the power curves are slightly larger when sample is of size 20.

Figures 6.1 - 11.3 show the comparisons between the actual power of the F-test compared to the H-test when samples are drawn from populations with different shapes. The comparisons are presented in the same procedure. It should be noted here that the power curves for the

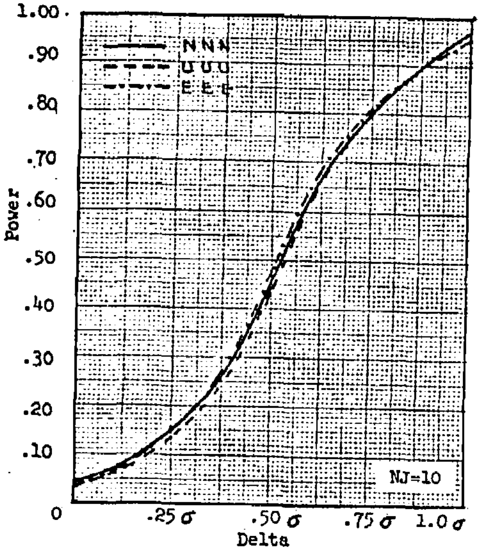

Figure 1.1

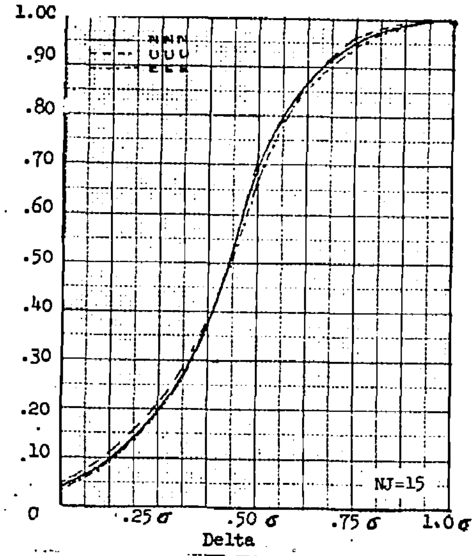

Figure 1.2

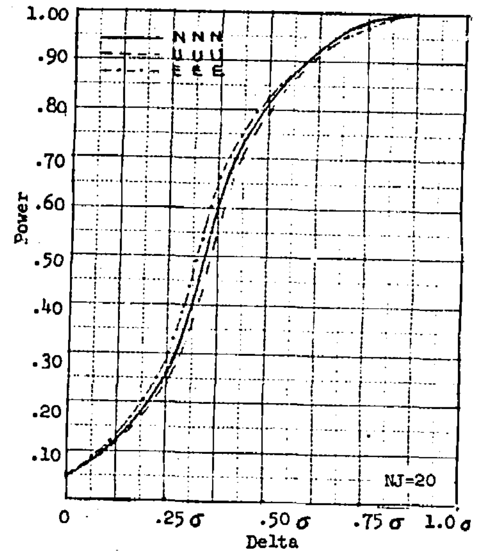

Figure 1.3

Figure 1.1,1.2, and 1.3 are the comparisons of power of F-(NNN) vs F-(UUU) vs F-(EEE) for NJ = 10, 15, and 20 respectively , $\alpha = .05$

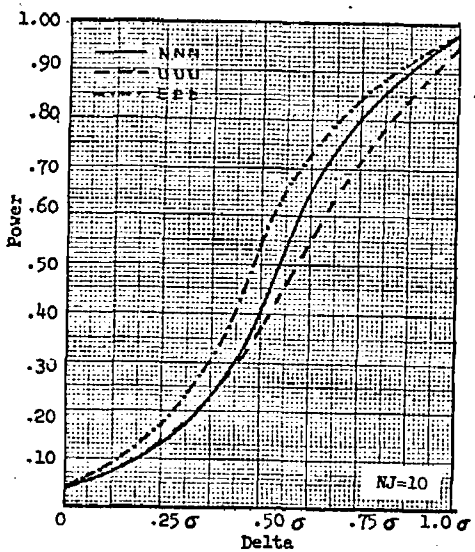

Figure 2.1

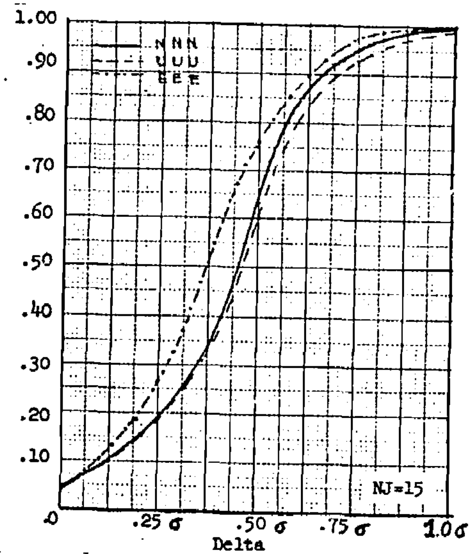

Figure 2.2

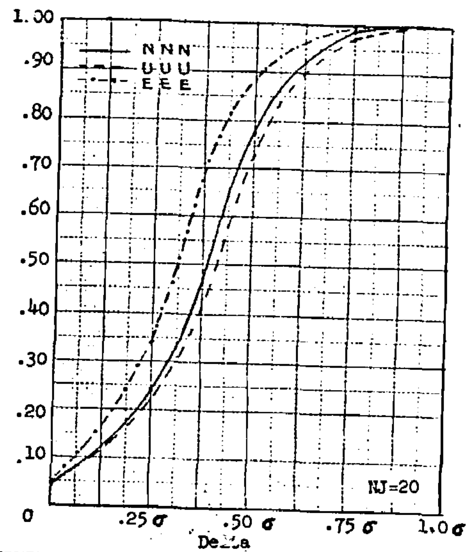

Figure 2.3

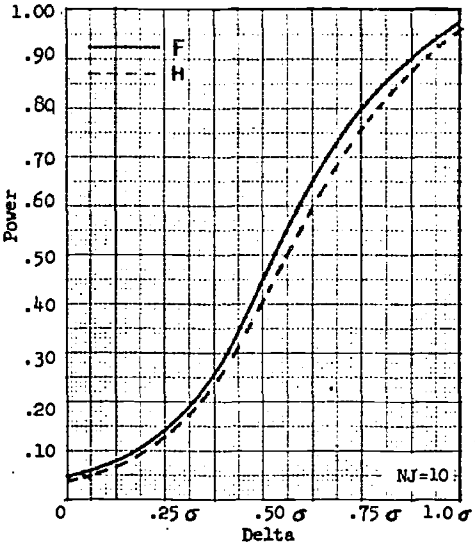

Figure3.1 Actual power of F-(NNN) vs H-(NNN) for NJ=10, nominal α =.05

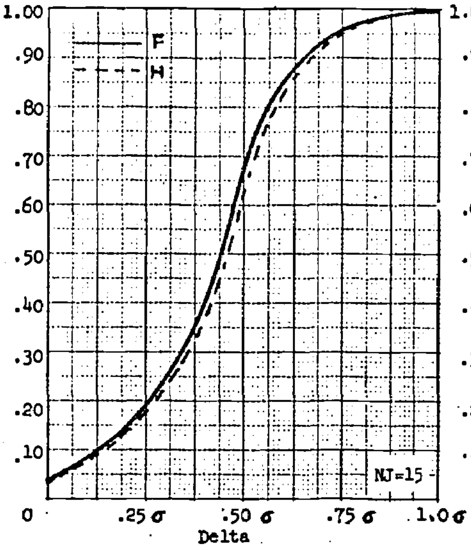

Figure 3.2 Actual power of F-(NNN) vs H-(NNN) for NJ=15, nominal α =.05

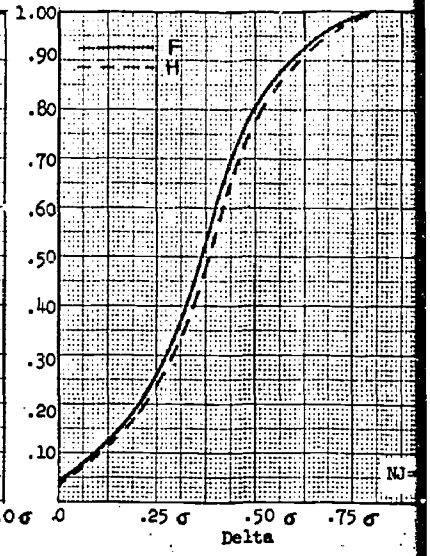

Figure 3.3 Actual power of F-(NNN)vs H-(NNN) for NJ=20, nominal α =.05

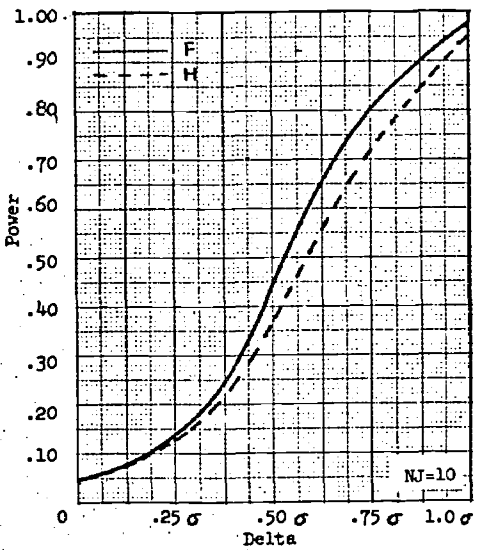

Figure 4.1 Actual power of F-(UUU) vs H-(UUU) for NJ=10, nominal α =.05

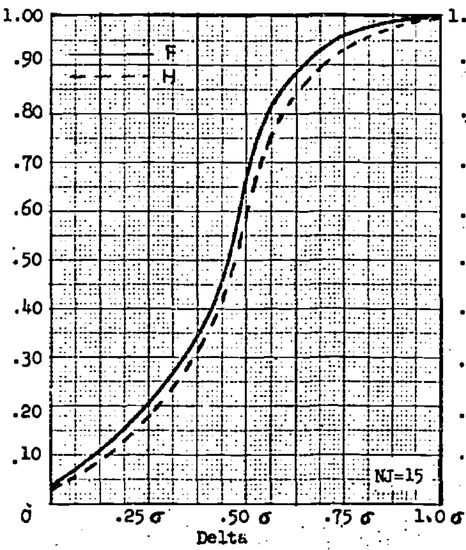

Figure 4.2 Actual power of F-(UUU) vs H-(UUU) for NJ=15, nominal α =.05

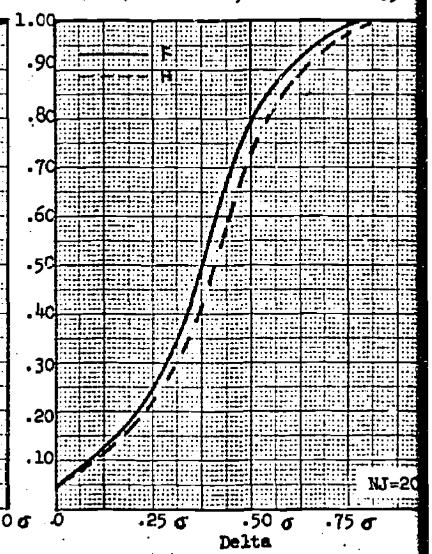

Figure 4.3 Actual power of F-(UUU) vs H-(UUU) for NJ=20, nominal α =.05

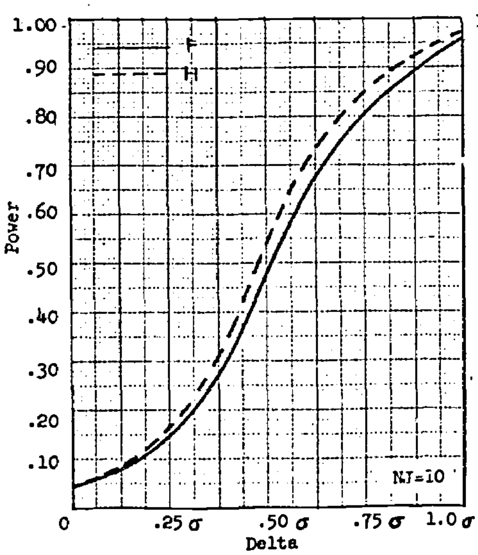

Figure 5.1 Actual power of F-(EEE) vs H-(EEE) for NJ=10, nominal α =.05

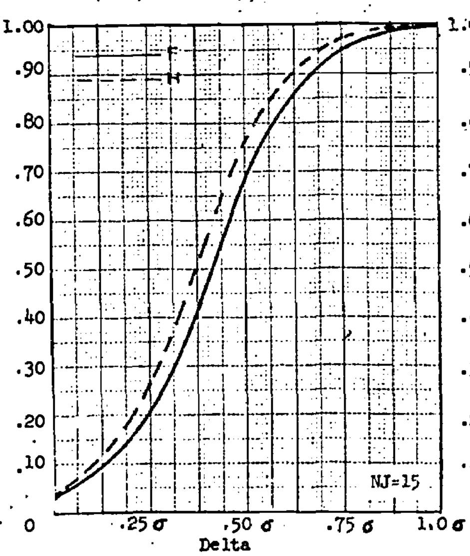

Figure 5.2 Actual power of F-(EEE) vs H-(EEE) for NJ=15, nominal α =.05

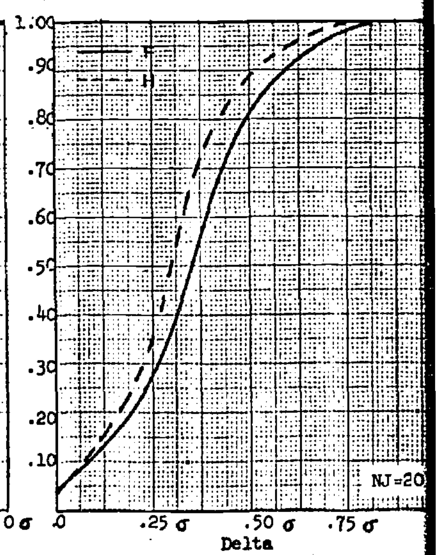

Figure 5.3 Actual power of F-(EEE) vs H-(EEE) for NJ =20, nominal α = .05

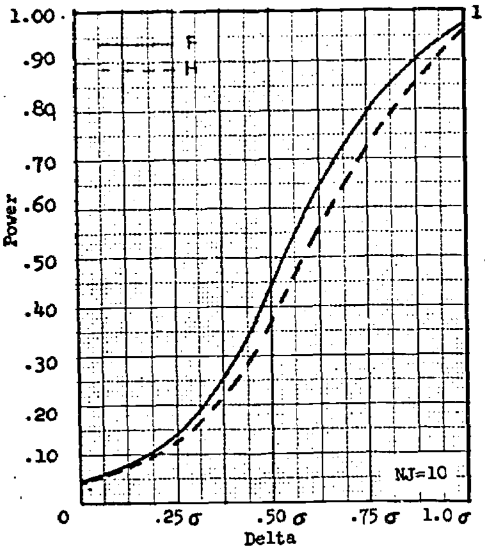

Figure 6.1 Actual power of F-(NNU) vs H-(NNU) for NJ=10, nominal α =.05

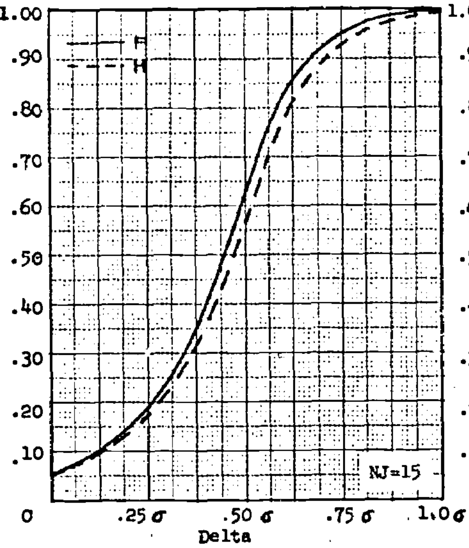

Figure 6.2 Actual power of F-(NNU) vs H-(NNU) for NJ=15, nominal α =.05

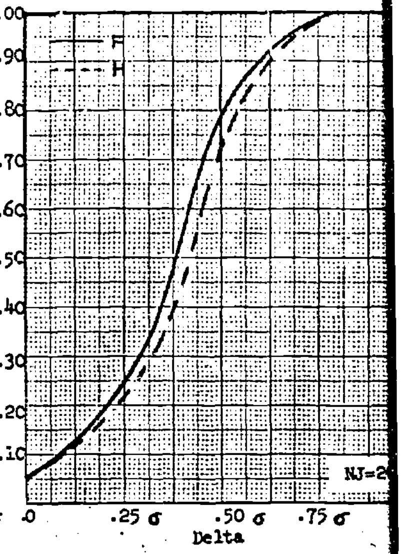

Figure 6.3 Actual power F-(NNU) vs H-(NNU) for NJ=20, nominal α =.05

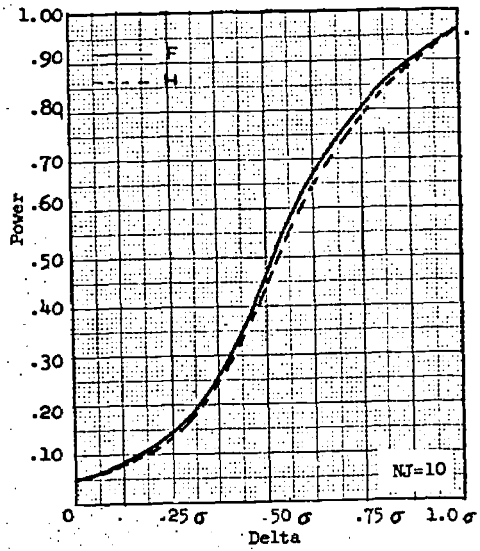

Figure 7.1 Actual power of F-(NNE) vs H-(NNE) for NJ=10, nominal α =.05

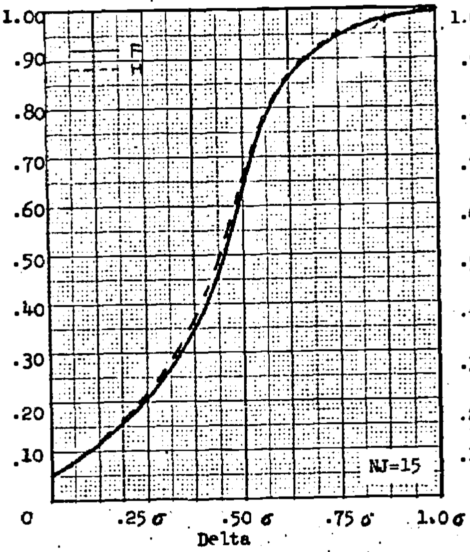

Figure 7.2 Actual power of F-(NNE) vs H-(NNE) for NJ=15, nominal α =.05

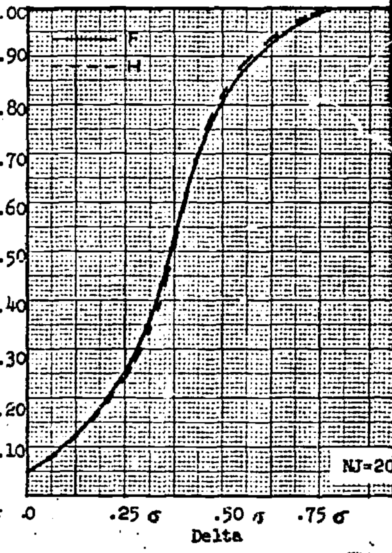

Figure7.3 Actual power of F-(NNE) vs H-(NNE) for NJ=20, nominal α =.05

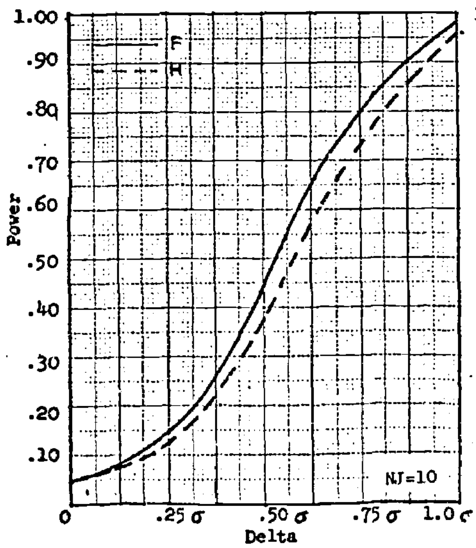

Figure 8.1 Actual power of F-(UUN) vs H-(UUN) for NJ=10, nominal α =.05

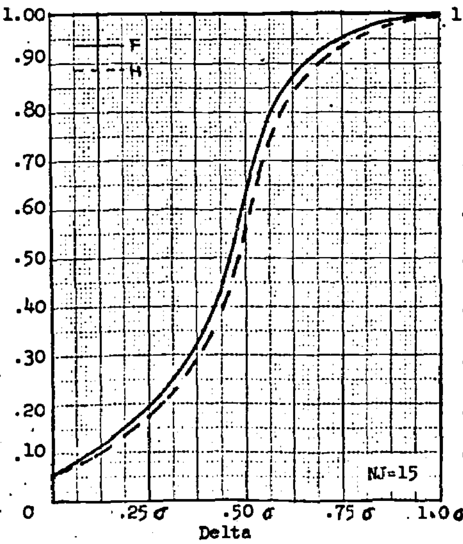

Figure 8.2 Actual power of F-(UUN) vs H-(UUN) for NJ=15, nominal α =.05

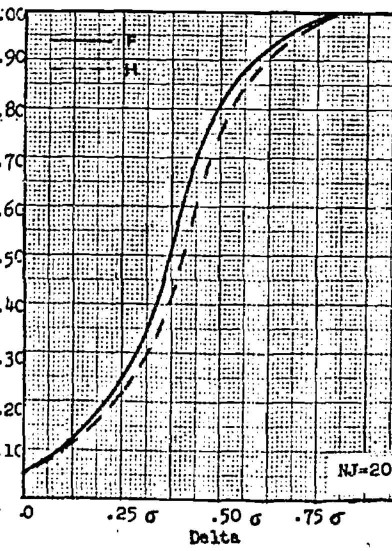

Figure 8.3 Actual power of F-(UUN) vs H-(UUN) for NJ=20, nominal α =.05

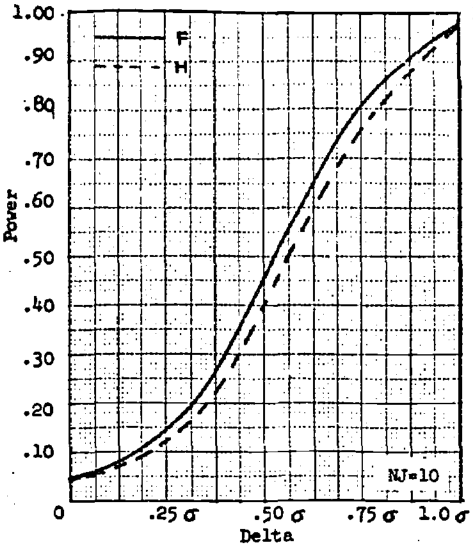

Figure 9.1 Actual power of F-(UUE) vs H-(UUE) for NJ=10, nominal α =.05

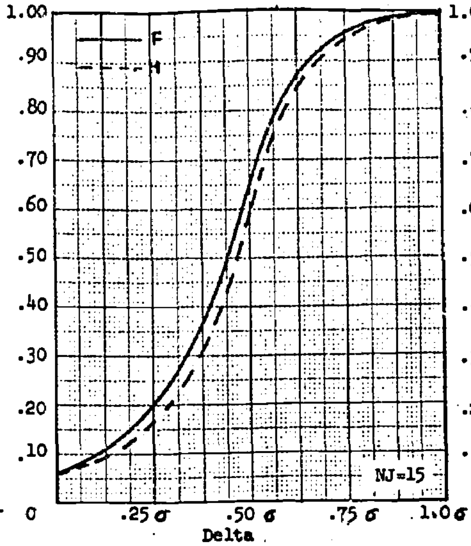

Figure 9.2 Actual power of F-(UUE) vs H-(UUE) for NJ=15, nominal α =.05

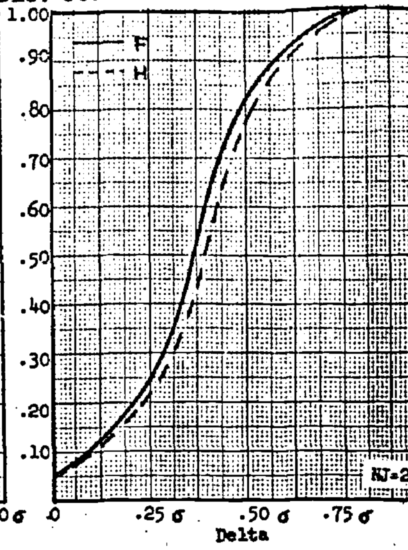

Figure 9.3 Actual power of F-(UUE) vs H-(UUE) for NJ=20, nominal α =.05

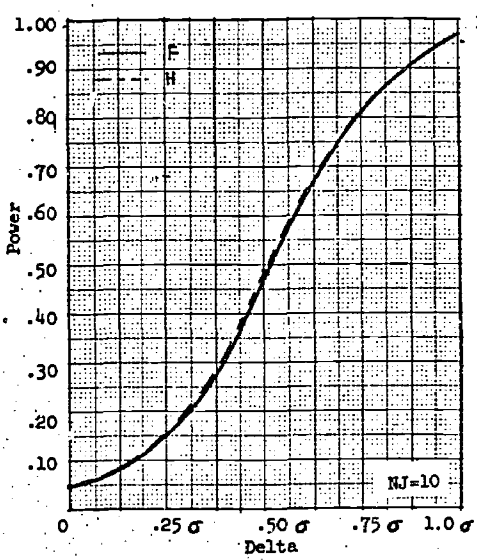

Figure 10.1 Actual power of F-(EEN) vs H-(EEN) for NJ=10, nominal α =.05

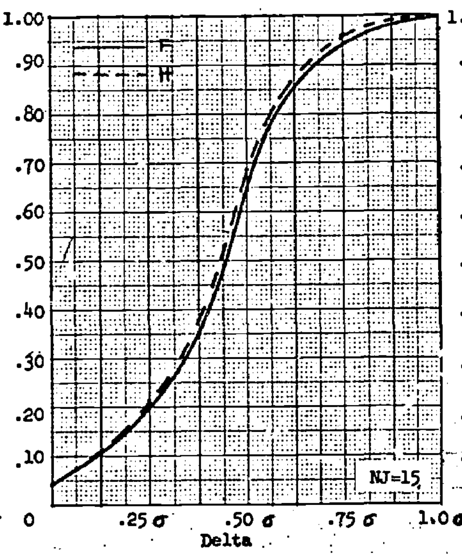

Figure 10.2 Actual power of F-(EEN) vs H-(EEN) for NJ=15, nominal α =.05

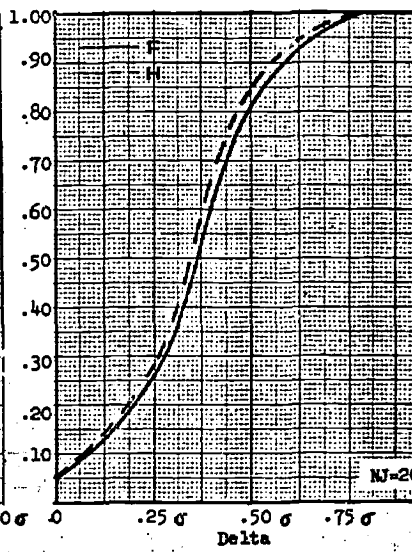

Figure 10.3 Actual power of F-(EEN) vs H-(EEN) for NJ=20, nominal α =.05

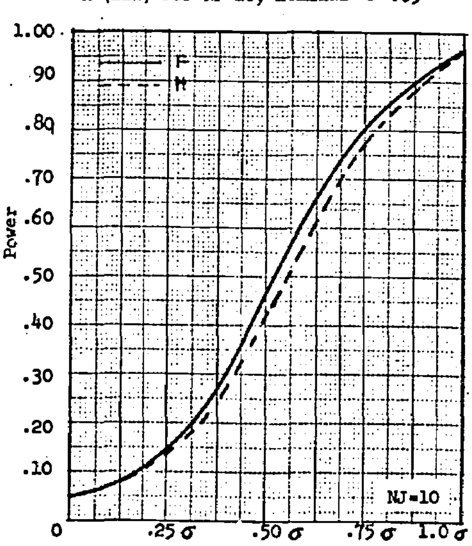

Figure 11.1 Actual power of F-(EEU) vs H-(EEU) for NJ=10, nominal α =.05

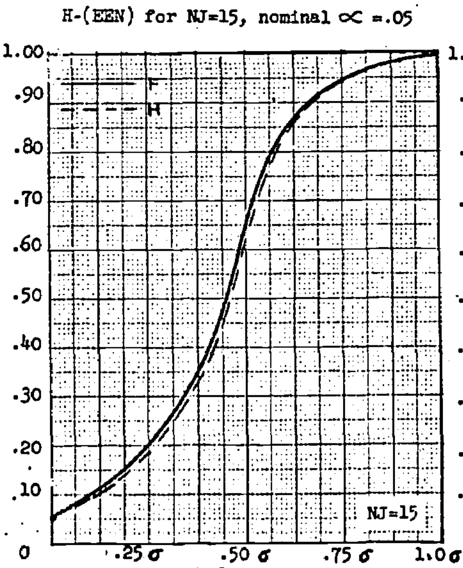

Figure 11.2 Actual power of F-(EEU) vs H-(EEU) for NJ=15, nominal α =.05

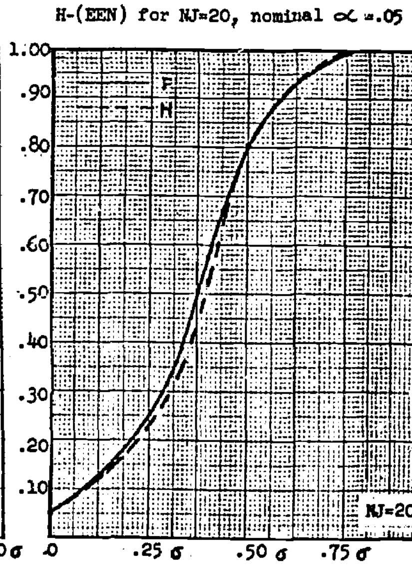

Figure 11.3 Actual power of F-(EEU) vs H-(EEU) for NJ=20, nominal α =.05

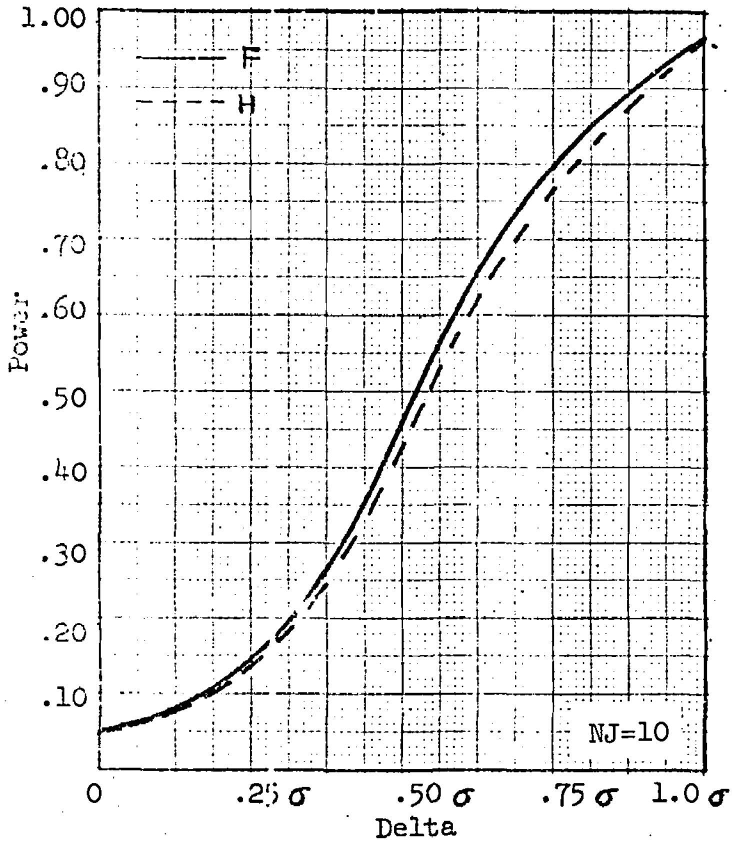

Figure 12.1 Actual power of F-(NUE) vs H-(NUE) for NJ=10, nominal α =.05

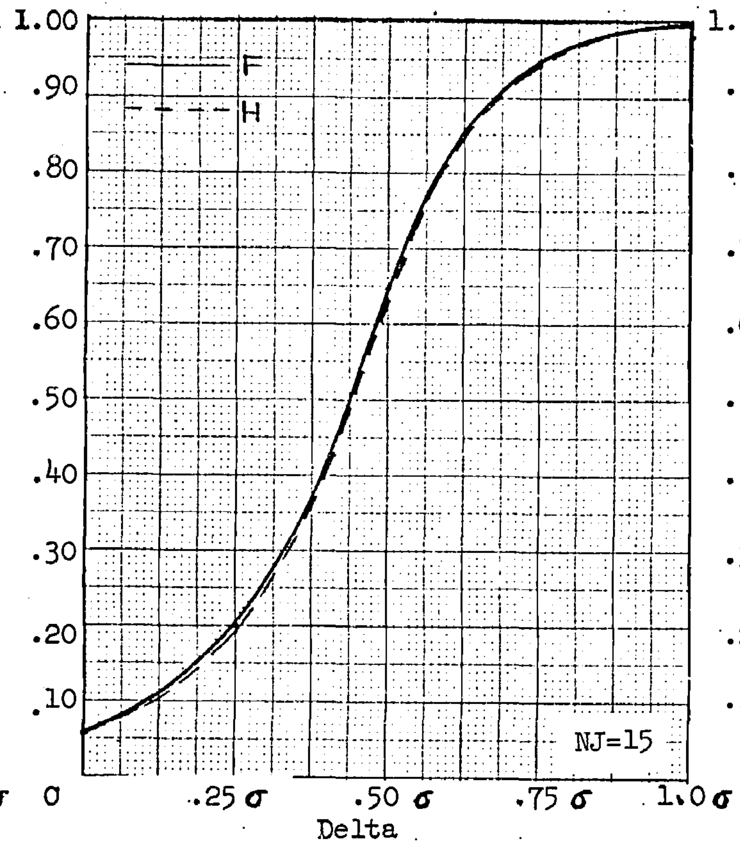

Figure 12.2 Actual power of F-(NUE) vs H-(NUE) for NJ=15, nominal α =.05

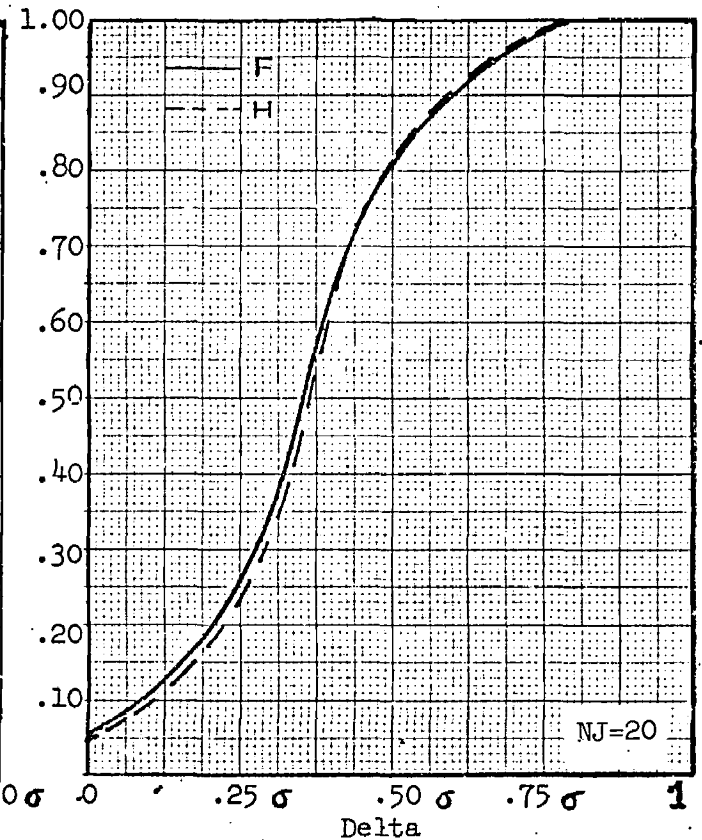

Figure 12.3 Actual power of F-(NUE) vs H-(NUE) for NJ=20, nominal α =.05

F-test and the H-test for sample size of 20 when delta is greater than .75 are extrapolated. This is because the powers of the F as well as the H are very close to 1.00 when delta is equal to .75 σ.

Figures 6.1, 6 2, and 6.3 show the power comparisons of F-(NNU) and H-(NNU) and for the sample sizes of 10, 15 and 20 respectively. It appears that the power of the F-test increases faster than the H-test for sample size 10, 15, and also 20. Their shapes are similar within the same sample size. The power comparisons of F-(UUN) versus H-(UUN) are as shown in figure 8.1, 8.2, and 8.3 . The results are the same as found for F-(NNU) versus H-(NNU).

Figures 7.1, 7.2, and 7.3 are the power comparisons of F-(NNE) versus H-(NNE) for sample size 10, 15 and 20 respectively. It is observed that the power curves of F-test and H-test are almost identical. The power of H-(NNE) is slightly less than F-(NNE) for sample size of 10, but the power of H-(NNE) is slightly greater than F-test when sample size is 15 and 20.

The powers of F-(UUE) versus H-(UUE) are shown in figure 9.1, 9.2 and 9.3 for sample of 10, 15, and 20 respectively. It is observed that the power of F-(UUE) is slightly greater than H-(UUE).

Figures 10.1, 10.2 and 10.3 show the power of F-(EEN) versus H-(EEN). The power of H-(EEN) is almost identical to F-(EEN) for sample of 10 but H-(EEN) has slightly greater power than F-(EEN) when sample sizes are 15 and 20.

The power comparisons of F-(EEU) with H-(EEU) are shown in figures 11.1, 11.2 and 11.3. It appears that when the sample size is 10, the power of H-(EEU) is slightly less than F-(EEU), but that comes very close together for sample sizes of 15 and 20. It is also observed that the power of H-(EEU) becomes slightly greater than F-(EEU) when delta is greater than .50.

Figures 12.1, 12.2 and 12.3 give almost the same picture as those shown in figures 11.1, 11.2 and 11.3. The power of H-(NUE, 15, .75) is .989 and F-(NUE, 15, .75) is .986. The power of (H-NUE) is slightly more than the power of F-(NUE) when the sample size is 20 and delta is greater than .50 σ

In summary, the power of the H-test is slightly greater than the F-test when the forms of population are EEE, and EEN, and also NNE for sample sizes of 15 and 20. The power of the H-test and the F-test are nearly identical when the population shapes are EEU, NUE, for sample of sizes 15 and 20. However, the power of H-test is slightly less than F-test when populations have the forms NNN, NNU, UUU, UUN, and UUE.

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